

Rotating Braneworld Black Holes

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We present a Kerr-Newman type stationary and axisymmetric solution that describes rotating black holes with a *tidal* charge in the Randall-Sundrum braneworld. The tidal charge appears as an imprint of nonlocal gravitational effects from the bulk space. We also discuss the physical properties of these black holes and their possible astrophysical appearance.

1. Introduction

The braneworld idea is a revolutionary idea to relate the properties of higher dimensional gravity to the observable world by direct probing of TeV-size mini black holes at high energy colliders. According to this idea our observable Universe is a slice, a "3-brane" in higher dimensional space.^{1,2} This in particular gives: (i) An elegant geometric resolution of the hierarchy problem between the electroweak scale and the fundamental scale of quantum gravity, (ii) the large size of the extra dimensions supports the weakness of Newtonian gravity on the brane and makes it possible to lower the scale of quantum gravity down to the electroweak interaction scale, (iii) the braneworld model (RS2 model) also supports the properties of four-dimensional Einstein gravity in low energy limit. In light of all this, it is natural to assume the formation of black hole in the braneworld due to gravitational collapse of matter trapped on the brane.

Several strategies have been discussed in the literature to describe the braneworld black holes. First of all, it has been argued that if the radius of the horizon of a black hole on the brane is much smaller than the size of the extra dimensions ($r_+ \ll L$), the black hole, to a good enough approximation, can be described by the usual classical solutions of higher dimensional vacuum Einstein equations. In the opposite limit when ($r_+ \gg L$), the black hole becomes effectively four-dimensional with a finite extension along the extra dimensions. The first simple solution pertinent to the latter case is based on the idea of a usual Schwarzschild metric on the brane that would look like a *black string* solution from the point of view of an observer in the bulk.³ However, the black string solution exhibits curvature singularities at infinite extension along the extra dimension.

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We shall discuss another strategy namely, we shall specify the metric form induced on the 3-brane assuming a Kerr-Schild ansatz for it. With this ansatz the system of the effective gravitational field equations on the brane^{4,5} becomes closed and the solution to this system turns out to be a Kerr-Newman type stationary axisymmetric black hole which possesses a *tidal* charge instead of a usual *electric* charge.

2. The metric form on the 3-brane

To describe a rotating black hole in the Randall-Sundrum scenario we shall make a particular assumption about metric on the brane, taking it to be of the Kerr-Schild form

$$ds^2 = (ds^2)_{flat} + H (l_i dx^i)^2, \quad (1)$$

where H is an arbitrary scalar function and l_i is a null, geodesic vector field in both the flat and full metrics. Earlier,⁶ this type of strategy was employed for a static black hole localized on the brane. With the metric form (1) the effective gravitational field equations on the brane

$$R_{ij} = -E_{ij}, \quad (2)$$

where E_{ij} the traceless "electric part" of the five-dimensional Weyl tensor, and the associated constraint equation

$$R = 0, \quad (3)$$

admit the solution which in the usual Boyer-Lindquist coordinates takes the form⁷

$$ds^2 = - \left(1 - \frac{2Mr - \beta}{\Sigma} \right) dt^2 - \frac{2a(2Mr - \beta)}{\Sigma} \sin^2 \theta dt d\phi \\ + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{2Mr - \beta}{\Sigma} a^2 \sin^2 \theta \right) \sin^2 \theta d\phi^2, \quad (4)$$

where

$$\Delta = r^2 + a^2 - 2Mr + \beta, \quad \Sigma = r^2 + a^2 \cos^2 \theta. \quad (5)$$

We see that that this metric looks exactly like the Kerr-Newman solution in general relativity, in which the square of the electric charge is "superceded" by a tidal charge parameter β . The Coulomb-type nature of the tidal charge is verified by calculating the components of the tensor E_{ij} through equation (2). Therefore one can think of it as carrying the imprints of nonlocal gravitational effects from the bulk space. Furthermore, the tidal charge may take on both *positive* and *negative* values.

3. Major Features

In complete analogy to the Kerr-Newman solution in general relativity, the metric (4) possesses two major features: The event horizon structure and the existence of a

static limit surface, the ergosphere. The event horizon is a null surface determined by the largest root of the equation $\Delta = 0$. We have

$$r_+ = M + \sqrt{M^2 - a^2 - \beta} \quad (6)$$

The horizon structure depends on the sign of the tidal charge. The event horizon does exist provided that

$$M^2 \geq a^2 + \beta. \quad (7)$$

Thus, for the positive tidal charge we have the same horizon structure as the usual Kerr-Newman solution. New interesting features arise when the tidal charge is taken to be negative. For $\beta < 0$ from equation (6) it follows that the horizon radius

$$r_+ \rightarrow (M + \sqrt{-\beta}) > M \quad (8)$$

as $a \rightarrow M$. This is not allowed in the framework of general relativity. From equations (6) and (7) it follows that for $\beta < 0$, the extreme horizon $r_+ = M$ corresponds to a black hole with rotation parameter a greater than its mass M . Thus, the bulk effects on the brane *may provide a mechanism for spinning up the black hole so that its rotation parameter exceeds its mass*. Meanwhile, such a mechanism is impossible in general relativity.

The static limit surface is determined by the equation $g_{tt} = 0$, the largest root of which gives the radius of the *ergosphere*

$$r_0 = M + \sqrt{M^2 - a^2 \cos^2 \theta - \beta}. \quad (9)$$

Clearly, this surface lies outside the event horizon coinciding with it only at angles $\theta = 0$ and $\theta = \pi$. The negative tidal charge tends to extend the radius of the ergosphere around the braneworld black hole, while the positive β just as the usual electric charge in the Kerr-Newman solution, plays the opposite role. For the extreme case, we find the radius of the ergosphere within

$$M < r < M + \sin \theta \sqrt{M^2 - \beta}. \quad (10)$$

We see that in astrophysical situations, the rotating braneworld black holes with negative tidal charge are more energetic objects in the sense of the extraction of the rotational energy from their ergosphere.

References

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